Key Engineering Materials Vols. 504-506 (2012) pp 77-82
Online available since 2012/Feb/03 at www.scientific.net
© (2012) Trans Tech Publications, Switzerland
doi:10.4028/www.scientific.net/KEM.504-506.77

Effect of the constitutive law on the accuracy of prediction of the forming limit curves

PARAIANU Liana[a], COMSA Dan Sorin[b], NICODIM Ioan[c], CIOBANU Ioan[d], BANABIC Dorel[e]

CERTETA - Research Centre, Technical University of Cluj-Napoca, Cluj-Napoca/Romania

[a] (liana.paraianu) [b] (dscomsa) [c] (ioan.nicodim) [d] (ioan.ciobanu) [e] (banabic)@tcm.utcluj.ro

Keywords: mechanical parameters, yield criterion, forming limit curves, Marciniak-Kuczynski model.

Abstract. The accuracy of the forming limit curves predicted by the Marciniak-Kuczynski model depends on the type and flexibility of the constitutive equations used to describe the mechanical response of the sheet metal. From this point of view, the yield criterion has the most significant influence. The paper presents a comparative analysis referring to the quality of the forming limit curves predicted by the Marciniak-Kuczynski model for the case when the plastic anisotropy of a DC04 sheet metal is described by the BBC2005 yield criterion. The coefficients included in the expression of the BBC2005 equivalent stress are evaluated using different identification strategies (with 4, 6, 7, and 8 mechanical parameters). The forming limit curves predicted by the Marciniak-Kuczynski model in each of the cases previously mentioned are compared with experimental data.

Introduction

The concept of Forming Limit Diagram (FLD) has been introduced by Keeler, Backofen and Goodwin [1, 2] on experimental bases, with the aim of characterizing the formability of sheet metals. The FLD is a curve relating pairs of principal limit strains, which can be obtained at the surface of the sheet metal during a forming process prior to the occurrence of some defects (necking, fracture, etc.). The first theoretical FLD models were based on the localized and diffuse necking theories proposed by Hill [3] and Swift [4], respectively. Since then, several mathematical models have been developed. One of these theories has been proposed by Marciniak and Kuczynski [5]. Even if this model was published in 1967, it remains the most popular model used for the calculation of the FLDs [6].

The FLD predictions are influenced by the shape of the yield locus used in the theoretical model. An analysis of this influence has been reported by Banabic [7] and Ahmadi [8]. A synthesis of the implementation of new constitutive equations in the models used for the computation of the limit strains is presented in Banabic et al. [9]. During the past five decades, the scientists have tried to develop new yield criteria able to describe the anisotropy of the sheet metals. The first anisotropic criterion was proposed by Hill in 1948 [10]. In 1979 [11] and 1990 [12] Hill improved the predictions of his first yield function. Another research direction leaded to yield criteria based on the crystallographic microstructure. These criteria have been proposed by Hosford [13] and improved by Barlat and Lian [14], Karafillis and Boyce [15], as well as by Barlat et al. [16] Banabic et al. [17] and Cazacu and Barlat [18] (see also the synthesis presented in [6] and [9]).

The yield criterion used in this paper belongs to the Hosford family. The basic formulation is BBC2005 [6] with eight coefficients. Four identification cases are taken into account in this work. The difference between them consists in the number of material parameters needed as input data (four, six, seven and eight different parameters, respectively).

The aim of this paper consists in evaluating the influence of the number of material parameters used in the identification on the determination of the forming limit curves. The predicted curves will be compared with experimental data. The yield loci calculated with different identification strategies of the BBC2005 yield criterion will be also presented.

All rights reserved. No part of contents of this paper may be reproduced or transmitted in any form or by any means without the written permission of TTP, www.ttp.net. (ID: 89.137.4.179-10/03/12,18:16:43)

Description of the BBC2005 yield criterion

The equivalent stress used in this study has the following formulation [6]:

$$\bar{\sigma}=\left[a\left(\Lambda+\Gamma\right)^{2k}+a\left(\Lambda-\Gamma\right)^{2k}+b\left(\Lambda+\Psi\right)^{2k}+b\left(\Lambda-\Psi\right)^{2k}\right]^{\frac{1}{2k}} \tag{1}$$

where $k \in N^{\geq 1}$ and $a,b > 0$ are material parameters, while Γ, Λ and Ψ are functions depending on the planar components of the stress tensor:

$$\Gamma=L\sigma_{11}+M\sigma_{22},\quad \Lambda=\sqrt{\left(N\sigma_{11}\text{-}P\sigma_{22}\right)^2+\sigma_{12}\sigma_{21}},\quad \Psi=\sqrt{\left(Q\sigma_{11}\text{-}R\sigma_{22}\right)^2+\sigma_{12}\sigma_{21}} \tag{2}$$

Nine material parameters are involved in the expression of the equivalent stress: k,a,b,L,M,N,P,Q and R. The integer exponent k depends on the crystallographic microstructure of the material: $k=3$ for BCC alloys and $k=4$ for FCC alloys. The stress components $\sigma_{\alpha\beta}\,(\alpha,\beta=1,2)$ are expressed in the plastic orthotropy frame (1 – rolling direction, 2 – transverse direction, 3 – normal direction).

The identification of this plasticity model can be performed using 4, 6, 7, or 8 mechanical parameters:

BBC2005 with 4 coefficients. In this case the following constraints are enforced:

$$L = N = Q,\ M = P = R \tag{3}$$

The identification procedure uses the uniaxial yield stress determined at 0^o and the anisotropy coefficients determined at 0^o, 45^o and 90^o from the rolling direction.

BBC2005 with 6 coefficients. In this case the following constraints are enforced:

$$L + M = 2N,\quad N = P \tag{4}$$

Six uniaxial parameters are used in the identification procedure: the uniaxial yield stresses and the anisotropy coefficients determined at 0^o, 45^o and 90^o from the rolling direction.

BBC2005 with 7 coefficients. In this case, only the first constraint given by Eq (4) must be enforced. Besides the uniaxial parameters, the identification uses the equibiaxial yield stress.

BBC2005 with 8 coefficients. In this case, the coefficients of the equivalent stress are computed using the uniaxial characteristics associated to the 0^o, 45^o and 90^o directions, as well as the yield stress and the anisotropy coefficient determined in equibiaxial tension.

Comparison of the yield loci obtained using different identification strategies of the BBC2005 criterion

The material used in this study is DC04 steel sheet with 0.85mm nominal thickness. Its mechanical parameters are listed in Tables 1 and 2. Table 1 shows the values of the uniaxial mechanical parameters: yield stresses and anisotropy coefficients determined on samples cut at 0^o, 45^o and 90^o from the rolling direction (n and K represent the coefficients in the Hollomon's hardening law). The values of yield stresses are normalized (divided by the yield stress associated to the 0^o direction). Table 2 lists the biaxial material characteristics. These are the yield stress and the anisotropy coefficient corresponding to the equbiaxial loading along the rolling and transverse directions.

As mentioned above, the yield criterion used in this study is BBC2005. Its identification can be performed if 4, 6, 7, or 8 mechanical parameters are available. Table 3 contains the values of the yield criterion coefficients corresponding to each of these identification cases.

Tabel 1 Uniaxial mechanical parameters of the DC04 steel (0.85 mm thickness)

Angle	r^{exp} [-]	$\sigma_\theta^{exp} / \sigma_0^{exp}$	n[-]	K[MPa]
0°	1.95	1.00	0.21	526.75
45°	1.29	1.06	0.20	541.323
90°	2.19	1.04	0.20	513.559

Table 2 Biaxial mechanical parameters of the DC04 steel (0.85 mm thickness)

$\sigma_b^{exp} / \sigma_0^{exp}$	r_b^{exp} [-]
1.28	0.84

Table 3 Coefficients of the BBC2005 yield criterion (DC04 steel; 0.85 mm thickness)

Yield criterion/Mechanical parameters	a	b	L	M	N	P	Q	R
BBC2005 with 4 coefficients	0.1506	0.3109	0.5687	0.5580	0.5687	0.5580	0.5687	0.5580
BBC2005 with 6 coefficients	0.3323	0.3930	0.5005	0.4767	0.4886	0.4886	0.6092	0.5594
BBC2005 with 7 coefficients	1.0123	0.3291	0.3590	0.3355	0.4903	0.4903	0.6209	0.5705
BBC2005 with 8 coefficients	1.0139	0.3290	0.3519	0.3422	0.4936	0.4864	0.6205	0.5717

Figure 1 shows a comparison of the yield loci predicted by different formulations of BBC2005. Three experimental points are also plotted on the same diagram. Due to the fact that both BBC2005 with 7 and 8 coefficients use in identification procedure the experimental value of σ_b^{exp}, the predictions of these formulations are more accurate.

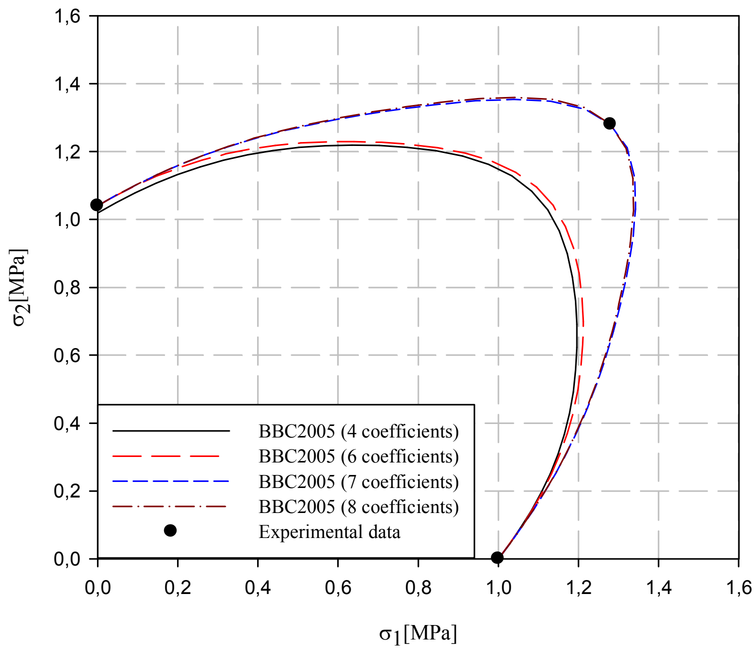

Fig. 1. Yield loci predicted for DC04 steel

Prediction of the Forming Limit Curves

The forming limit curves have calculated using the Marciniak-Kuczynski (M-K) model (Fig. 2).

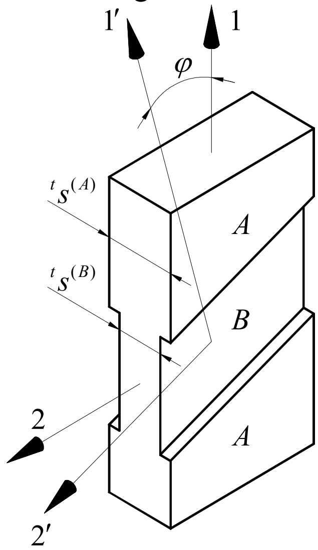

Fig. 2. Marciniak-Kuczinski model of a geometric imperfection

The model is based on the assumption of a geometrical inhomogeneity already existing in the material. According to this hypothesis, a narrow strip with different thickness from the nominal one is present. In the fig. 2, the region denoted with B is the thinner one, while the region A has a nominal thickness:

$$ {}^{t}f = {}^{t}s^{(B)} \big/ {}^{t}s^{(A)}, \quad 0 < {}^{t}f < 1 \tag{5} $$

The parameter ${}^{t}f$ is the so-called inhomogeneity factor and represents the ratio of the current thicknesses ${}^{t}s^{(A)}$ and ${}^{t}s^{(B)}$ associated to the normal and defective regions of the sheet metal, respectively. In this paper, the inhomogeneity factor has been set to the initial value ${}^{0}f = 0.9995$.

An implicit numerical scheme has been developed for solving the M-K model [6]. Due to the fact that the sheet metal is considered to behave as an orthotropic membrane under plane-stress conditions, the following constrains for both regions should be imposed:

$$ \begin{aligned} {}^{t}\sigma_{i3} &= {}^{t}\sigma_{3i} = 0, \quad i = 1,2,3, \\ {}^{t}\dot{\varepsilon}_{\alpha 3} &= {}^{t}\dot{\varepsilon}_{3\alpha} = 0, \quad \alpha = 1,2,3. \end{aligned} \tag{6} $$

where ${}^{t}\sigma_{ij}$ and ${}^{t}\varepsilon_{ij}$ are the stress and strain-rate tensors expressed in the plastic orthotropy frame. The orientation of the groove is given by the angular parameter φ. In the tension-tension region of the forming limit band the unfavorable condition is obtained when the groove is perpendicular to the tensile direction. In the case of the left region, the inclination of the necking band is given by the following formula:

$$ \varphi = \arctan\sqrt{\max\left[-\rho^{(A)}, 0\right]}, \quad -1 < \rho^{(A)} \le 1 \tag{7} $$

where $\rho^{(A)} = \dot{\varepsilon}_2^{(A)} \big/ \dot{\varepsilon}_1^{(A)} = const.$ is the the strain-rate ratio in region A. One may notice that Eq. (7) is similar to the one found by Hill [3].

The following relationships express the continuity of the strain-rate along the necking band:

$$ {}^{t}\dot{\varepsilon}_{2'2'}^{(A)} = {}^{t}\dot{\varepsilon}_{2'2'}^{(B)} \tag{8} $$

The mechanical equilibrium at the interface of regions A and B is described by the equations

$$
\begin{aligned}
{}^t\sigma_{1'1'}^{(A)} \cdot {}^t s^{(A)} &= {}^t\sigma_{1'1'}^{(B)} \cdot {}^t s^{(B)} \\
{}^t\sigma_{1'2'}^{(A)} \cdot {}^t s^{(A)} &= {}^t\sigma_{1'2'}^{(B)} \cdot {}^t s^{(B)}
\end{aligned}
\tag{9}
$$

The implicit scheme allows the reduction of the M-K model to the numerical solution of a single non-linear equation [6]. In order to avoid any divergence, the authors have used the bisection method coupled with a bracketing strategy.

Fig. 3 shows the forming limit curves predicted by the M-K model for different identification cases of the BBC2005 yield criterion (with 4, 6, 7 and 8 mechanical parameters). Some experimental points are also placed on the diagram. One may notice that the best predictions of the M-K model correspond to the cases when the BBC2005 yield criterion has been identified with 7 and 8 parameters. In both situations, the calculated curves are in the vicinity of the experimental points. If only 4 or 6 mechanical parameters are used in the identification procedure, the predictions of the M-K model overestimate the formability along the right branch of the FLD (tension-tension). The differences between the calculated limit curves can be easily explained by making reference to the shape of the yield loci shown in Fig. 1. In the cases when the identification procedure is based on 7 or 8 mechanical parameters, the distance between the plane strain and the equibiaxial point is shorter. As a consequence, the strain localization process will be accelerated and the right branch of the FLD will be lowered. Fig. 1 also shows that the equibiaxial yield stress has a considerably stronger influence on the shape of the yield locus than the equibiaxial coefficient of plastic anisotropy. Due to this fact, the forming limit curves predicted by the BBC2005 identified with 7 and 8 mechanical parameters are almost coincident.

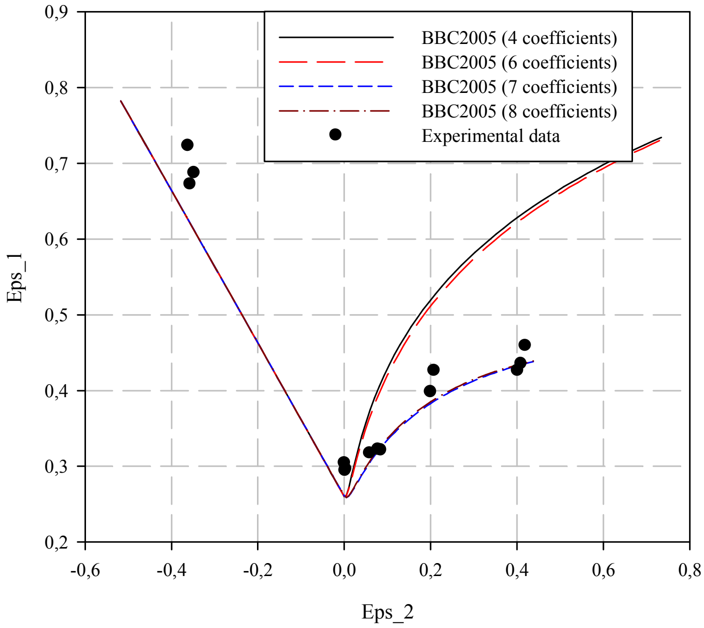

Fig. 3. Forming limit curve predicted with Marciniak-Kuczynski model for DC04 steel

Conclusions

The aim of this paper consists in evaluating the influence of the identification procedure of the BBC2005 yield criterion (use of different sets of mechanical parameters as input data) on the quality of the forming limit curves predicted by the M-K model. The analysis performed by the authors in the case of a DC04 steel sheet reveals a close relationship between the shape of the yield locus and the limit strains resulted from calculations. The interconnection is very strong at the level of the right branch of the FLD. The accuracy of the results provided by the M-K model can be considerably improved if the plasticity of the sheet metal is described by yield criteria that use equibiaxial mechanical parameters in their identification procedure. From these parameters, the equibiaxial yield stress seems to have the most important influence both on the shape of the yield locus and the limit strains corresponding to the tension-tension branch of the FLD.

Acknowledgments

This paper was supported by the following projects: POSDRU/89/1.5/S/52603, PN II-RU-615/210/2010, POSDRU/107/1.5/S/78534 and PCCE 100/2010.

References

[1] S.P. Keeler and W.A. Backofen, Plastic instability and fracture in sheets stretched over rigid punches, ASM Trans. 56 (1963) 25-48.

[2] G.M. Goodwin, Application of strain analysis to sheet metal forming problems in the press shop, SAE Automotive Engineering Congress, (1968) paper no. 680093.

[3] R. Hill, On discontinuous plastic states, with special reference to localized necking in thin sheets, J. Mech. Phys. Solids 1 (1952) 19–30.

[4] H.W. Swift, Plastic instability under plane stress, J. Mech. Phys. Solids 1 (1952) 1–18.

[5] Z. Marciniak and K. Kuczynski, Limit strains in the processes of stretch forming sheet metal, Int. J. Mech. Sci. 9 (1967) 609–612.

[6] D. Banabic, Sheet Metal Forming Processes. Constitutive modelling and numerical simulation, Springer, Heidelberg, Berlin, 2010.

[7] D. Banabic, E. Dannenmann, The influence of the yield locus shape on the limits strains, J. Mat. Process. Technol. 109 (2001) 9-12.

[8] S. Ahmadi, A.R. Eivani, A. Akbarzadeh, An experimental and theoretical study on the prediction of forming limit diagrams using new BBC yield criteria and M–K analysis, Computational Materials Science 44 (2009) 1272-1280

[9] D. Banabic, F. Barlat, O. Cazacu, T. Kuwabara, Advances in anisotropy and formability, International Journal of Material Forming 3 (2010) 165-189.

[10] R. Hill, The mathematical theory of plasticity, Clarendon Press, Oxford, 1950.

[11] R. Hill, Theoretical plasticity of textured aggregates, Math. Proc. Cambridge Phil. Soc. 85 (1979) 179-191.

[12] R. Hill, Constitutive modelling of orthotropic plasticity in sheet metals, J. Mech. Phys. Solids 38 (1990) 405-417.

[13] W.F. Hosford, On yield loci of anisotropic cubic metals, Proc. 7th North American Metalworking Conference, Dearborn (1979) 191-197.

[14] F. Barlat, J. Lian, Plastic behaviour and stretchability of sheet metals. Part I: A Yield Function for Orthotropic Sheets under Plane Stress Conditions, Int. J. Plasticity 5 (1989) 51-66.

[15] A.P. Karafillis, M.C.Boyce, A general anisotropic yield criterion using bounds and a transformation weighting tensor, J. Mech. Phys. Solids 41(1993) 1859-1886.

[16] F. Barlat et al., Plane stress yield function for aluminium alloy sheets – Part 1: Theory, Int. J. Plasticity 19 (2003) 297–319.

[17] D. Banabic et al., An improved analytical description of orthotropy in metallic sheets, Int. J. Plasticity 21 (2005) 493–512.

[18] O. Cazacu, F. Barlat, Generalization of Drucker's yield criterion in orthotropy, Mathematics and Mechanics of Solids 6 (2001) 613–630.

Material Forming ESAFORM 2012
10.4028/www.scientific.net/KEM.504-506

Effect of the Constitutive Law on the Accuracy of Prediction of the Forming Limit Curves
10.4028/www.scientific.net/KEM.504-506.77