Influence of the Constitutive Equations on the FLC Prediction

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The forming limit curve (FLC) is the most popular procedure used for determining the formability of the sheet metals. The main objective of this work is to establish the influence of the mechanical parameters on the predicted FLC. This study is focused on determining the influence of the mechanical parameters in three regions of the FLC: tension-compression, plane-strain and tension-tension. The material used in the analysis is DC04 steel sheet with 0.85 mm thickness. The limit strains are calculated on the basis of the Marciniak-Kuczinsky necking criterion. In order to determine the influence of the mechanical parameters on the FLC prediction the Design of Experiments (DOE) approach has been used. By applying the ANOVA procedure, the influence of each factor on the limit strains in the regions mentioned above is evaluated. The mechanical behavior of the material has been described by the classical Hill48 yield criterion and the non-quadratic BBC2005 yield criterion. Finally, a comparison of the results obtained for each yield criterion is presented.

Keywords: Forming limit band, Sensitivity, Design of Experiment, ANOVA

1. INTRODUCTION

The FLC concept has been introduced for the first time by Keeler and Backofen[1] and Goodwin[2], respectively. Due to its simplicity and efficiency, the FLC has been rapidly adopted by the industry.

During the second half of the 20th century, several models for the calculation of the limit strains have been developed. Even if the Marciniak-Kuczynsky (M-K) model[3] was published in 1967, it remains the most popular model used for the calculation of the FLCs. The strain localization phenomenon has been modeled by making the assumption that a thickness inhomogeneity exists from the very beginning of the forming process. In its original formulation, this model can be used only for calculating the right branch of the FLC. In order to extend the applicability of the M-K model to the tension-compression region of the FLC, Hutchinson and Neale[4] have developed a more general formulation that allows the planar rotation of the thickness defect. A review of the Marcinik-Kuckzynski model is presented in Banabic and Dorr[5], Banabic et al.[6], Banabic[7], Banabic et al.[8]. An exhaustive description of the experimental and theoretical research on FLC's can be found in Banabic[9]. The first who noticed the variability of the experimentally determined FLC's were van Minh[10]. Based on their observations, Janssens et al.[11] introduced the more general concept of Forming Limit Band (FLB). By taking into account the variability of the mechanical parameters, lower and upper FLC's of the sheet metals can be drawn.

The main purpose of this work is to analyze the influence of the mechanical parameters on the uniaxial, plane strain and biaxial regions of the forming limit curve. By using the design of experiment methodology[11] together with the ANOVA method, the percent contribution of the mechanical parameters on the FLC has been calculated. The mechanical behavior of the material is described by Hill48 and BBC2005 yield criteria, respectively. The hardening is described by Swift's law. The FLB obtained in both cases is then analyzed.

2. NECKING CRITERION USED FOR THE DETERMINATION OF THE FORMING LIMIT CURVE

In order to calculate the FLC, an implicit method based on M-K necking model has been developed[7]. As it has been mentioned above, the model is based on the assumption that a thinner region preexists on the surface of the sheet metal (figure 1).

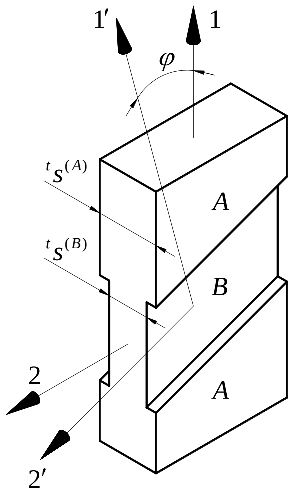

Fig.1. Marciniak-Kuczinsky model

The region having the nominal thickness is denoted as A in figure 1, while the defective strip having a smaller thickness is denoted as B. The current amplitude of the thickness defect is described by the quantity:

$$ {}^{t}f = {}^{t}s^{(B)} / {}^{t}s^{(A)}, \quad 0 < {}^{t}f < 1 \tag{1} $$

The parameter ${}^{t}f$ is the so-called non-homogeneity factor and represents the ratio of the current thicknesses ${}^{t}s^{(A)}$ and ${}^{t}s^{(B)}$ associated to the normal and defective regions of the sheet metal, respectively. In this paper, the non-homogeneity factor has the initial value equal to 0.999.

In order to determine the limit strains in tension-tension region, the groove is orientated perpendicular to the tensile direction. In the case of the left branch of the FLC, the inclination of the necking band given by the angular parameter φ has the following formula:

$$ \varphi = \arctan\sqrt{\max\left[-\rho^{(A)},0\right]}, \quad -1 < \rho^{(A)} \le 1 \tag{2} $$

where $\rho^{(A)} = \dot{\varepsilon}_2^{(A)} / \dot{\varepsilon}_1^{(A)} = const.$ is the strain-rate ratio associated to region A. The following relationship expresses the continuity of the strain-rate:

$$ {}^{t}\dot{\varepsilon}_{2'2'}^{(A)} = {}^{t}\dot{\varepsilon}_{2'2'}^{(B)} \tag{3} $$

The mechanical equilibrium at the interface of regions A and B is described by the equations

$$ \begin{aligned} {}^{t}\sigma_{1'1'}^{(A)} \cdot {}^{t}s^{(A)} &= {}^{t}\sigma_{1'1'}^{(B)} \cdot {}^{t}s^{(B)} \\ {}^{t}\sigma_{1'2'}^{(A)} \cdot {}^{t}s^{(A)} &= {}^{t}\sigma_{1'2'}^{(B)} \cdot {}^{t}s^{(B)} \end{aligned} \tag{4} $$

The implicit scheme allows the reduction of the M-K model to the numerical solution of a single non-linear equation[7]. In order to avoid any divergence, the authors have solved this equation using the bisection method coupled with a bracketing strategy.

3. MATERIAL CHARACTERIZATION

The material used in this study is DC04 steel sheet with 0.85mm thickness. The tensile tests have been performed on samples cut at 0^o, 45^o and 90^o from the rolling direction. The yield stresses and the anisotropy coefficients have been determined for each direction. The hardening exponent coefficient of Swift's hardening law (*n*) has been determined only on samples cut at 0^o with respect to the rolling direction. The standard deviation of each mechanical parameter has been calculated by taking into account more than 30 experimental results. The number of the experiments has been chosen according to Janssens et al.[9] for a confidence level of 99.5% for Gauss normal distribution.

An important factor that characterizes the material is the equibiaxial yield stress. This parameter is determined using a hydraulic bulge test. Due to the fact that its determination is rather difficult, the standard deviation of this parameters has been set equal to the standard deviation of the yield stress determined along the rolling direction.

Table 1 shows the mean values of the mechanical parameters obtained from experiments, together with their standard deviations.

Table.1. Mechanical parameters of the DC04 steel sheet (0.85 mm thickness).

Material parameter	Mean value	Standard deviation
n_0	0.21	0.002
Y_0[MPa]	195.96	2.086
r_0	1.92	0.110
Y_{45}[MPa]	210.97	2.401
r_{45}	1.31	0.062
Y_{90}[MPa]	205.49	2.154
r_{90}	2.22	0.145
Y_b[MPa]	249.72	2.086

4 DESIGN OF EXPERIMENT

In order to determine the influence of each parameter on the FLC, the number of numerical simulations has to be established. The Taguchi design of experiments method has been used to analyze the interaction effect between various controllable factors. In this analysis, Taguchi method has been used in order to investigate the effects and interactions of eight noise variables. All these parameters are assumed to obey Gauss normal distribution. Due to this fact, two levels of the mechanical parameters have been established (see table 2). The first level is calculated by subtracting 3Sigma from the mean value, while the second level results by adding 3Sigma to the mean value.

Table.2. Mechanical parameters and their reference levels.

Mechanical parameters	Level 1	Level 2
n	0.2037	0.216
Y_0[MPa]	189.7	202.22
r_0	1.59	2.25
Y_{45}[MPa]	203.77	218.18
r_{45}	1.13	1.51
Y_{90}[MPa]	199.03	211.96
r_{90}	1.78	2.66
Y_b[MPa]	243.462	255.978

By using the levels of the mechanical parameters an orthogonal array with combinations of possible conditions has been constructed. In this study two yield criteria are used in order to describe the mechanical behavior of the material: Hill48 yield criterion and the non-quadratic BBC2005 yield criterion. The identification procedure of Hill48 yield criterion uses only four mechanical parameters, while BBC2005 yield criterion needs for calculating its coefficients seven mechanical parameters. To determine the influence of the mechanical parameters on the FLC by using Hill48 yield criterion, an L8 orthogonal array has been chosen in order to establish the input of the numerical simulations (table 3). The response of the constitutive model has been listed in the last three columns of the table.

Table.3. L8 orthogonal array and the model response for three regions of limit strains (BT – biaxial traction, PS – plane strain, UT – uniaxial traction).

Nr	n_0	Y_0	r_0	r_{45}	r_{90}	BT	PS	UT
1	1	1	1	1	1	0.646	0.176	0.467
2	1	1	1	2	2	0.585	0.176	0.467
3	1	2	2	1	1	0.558	0.176	0.588
4	1	2	2	2	2	0.498	0.177	0.588
5	2	1	2	1	2	0.508	0.188	0.626
6	2	1	2	2	1	0.569	0.188	0.626
7	2	2	1	1	2	0.596	0.188	0.497
8	2	2	1	2	1	0.657	0.188	0.497

In the case when using BBC2005 yield criterion, an L12 orthogonal array has been constructed (table 4). As in the case of Hill48, the last three columns of table 4 are occupied by the calculated limit strains. In both cases, the exponent of the Swift hardening law has been taken into account.

Table.4. L12 orthogonal array and the model response for three regions of limit strains (BT – biaxial traction, PS – plane strain, UT – uniaxial traction).

Nr	n_0	Y_0	r_0	Y_{45}	r_{45}	Y_{90}	r_{90}	Y_b	BT	PS	UT
1	1	1	1	1	1	1	1	1	0.309	0.174	0.465
2	1	1	1	1	1	2	2	2	0.307	0.174	0.465
3	1	1	2	2	2	1	1	1	0.340	0.174	0.586
4	1	2	1	2	2	1	2	2	0.332	0.173	0.465
5	1	2	2	1	2	2	1	2	0.354	0.174	0.586
6	1	2	2	2	1	2	2	2	0.434	0.174	0.586
7	2	1	2	2	1	1	2	1	0.312	0.186	0.626
8	2	1	2	1	2	2	2	1	0.415	0.187	0.626
9	2	1	1	2	2	2	1	2	0.307	0.186	0.497
10	2	2	2	1	1	1	1	2	0.321	0.186	0.626
11	2	2	1	2	1	2	1	1	0.386	0.186	0.497
12	2	2	1	1	2	1	2	1	0.397	0.186	0.497

5. RESULTS AND DISSCUSSIONS

The aim of this study is to analyze the influence of the mechanical parameters on the forming limit curve. This was possible by using the analysis of the variance (ANOVA) method.

The study reveals the influence of the mechanical parameters on three regions of the forming limit curve: biaxial traction (BT), plane strain (PS) and uniaxial traction (UT). The influence is quantified in percents. Table 5 shows the importance degree of each parameter on the forming limit curve if the plasticity of the sheet metal is described by Hill48 yield criterion. If the non-quadratic yield criterion BBC2005 is used, the influence distributions of each mechanical parameter on the limit strains are listed in table 6.

Table.5. Percent influence of each mechanical parameter on the limit strains corresponding to the biaxial, plane strain and uniaxial regions of the FLC predicted by BBC2005 yield criterion

Material parameter	BT%	PS%	UT%
n_0	1.283	99.746	7.682
Y_0	19.809	0.0003	0.000
r_0	6.835	0.166	92.199
Y_{45}	-0.0528	0.0008	0.000
r_{45}	2.035	0.00003	0.000
Y_{90}	13.380	0.054	0.000
r_{90}	11.915	-0.0001	0.000
Y_b	43.911	0.030	0.000
Error -other parameters	0.8848	0.00297	0.119

Table.6. Percent influence of each mechanical parameter on the limit strains corresponding to the biaxial, plane strain and uniaxial regions of the FLC predicted by Hill48 yield criterion

Material parameter	BT%	PS%	UT%
n_0	1.012	99.8266	6.903
Y_0	-0.001	-0.0004	0.087
r_0	66.738	0.0913	93.01
r_{45}	-0.0003	-0.0006	0.000
r_{90}	32.2438	0.0789	0.000
Error -other parameters	0.0075	0.0042	0

As one may notice, the hardening exponent has the strongest influence on the plane-strain region in both cases (about 99%). Due to the fact that the number of mechanical parameters is not equal in the studied cases, the percent contributions of each mechanical parameter on the limit strains corresponding to the biaxial, plane strain and uniaxial regions of the FLC are different. In the compression - tension area the most influent mechanical parameter is the anisotropy coefficient determined on samples cut along the rolling direction. This is valid both for Hill48 and BBC2005 yield criteria. But in the tension - tension quadrant the most important effect on the limit strains is not given by the same parameters for both yield criteria. More precisely, if Hill48 yield criterion is used, the most influent parameters are the coefficients of plastic anisotropy r_0 and r_{90}: about 66% and 32%, respectively. In the case of the BBC2005 yield criterion, the most influent parameters are the equibiaxial yield stress and the uniaxial yield stress determined at 0^o from the rolling direction: about 44% and 20%, respectively.

The forming limit curves are strongly influenced by the shape of the yield loci. The dispersion observed in the yield loci based on the scattering of the mechanical parameters used in the identification procedure is reflected by the predicted forming limit curves. The influence of the mechanical parameters on the yield locus described by BBC2005 and Hill48 models, respectively is shown in figure 2 and figure 3. The simulations have been performed according to the design of experiments method. Due to the fact that the yield locus predicted by Hill48 is controlled by a less number of parameters than in the case of BBC2005, the scattering observed in its prediction is larger. The larger scattering noticed in the biaxial region is a consequence of the fact that Hill48 identification procedure uses only the uniaxial material data.

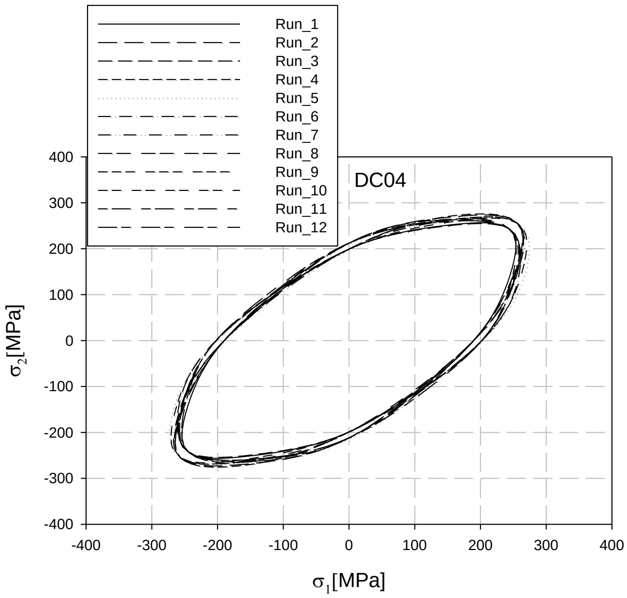

Fig.2. Yield loci predicted by BBC2005 yield criterion according to the design of experiments

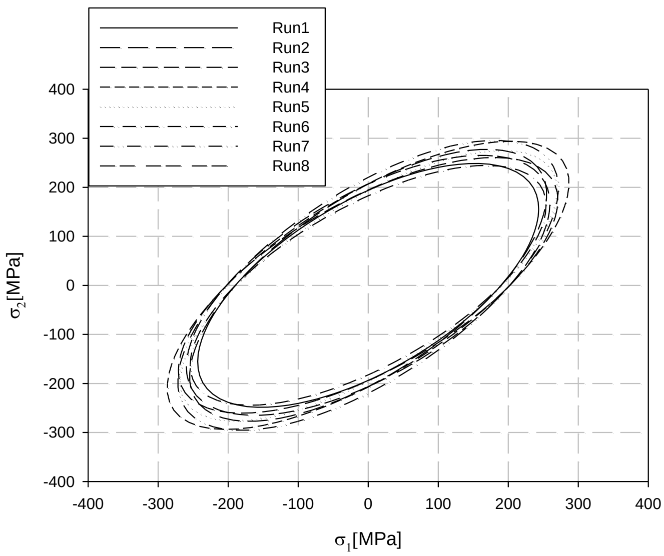

Fig.3. Yield loci predicted by Hill48 yield criterion according to the design of experiments

The forming limit curves calculated by varying the mechanical parameters according to the design of experiment methodology are presented in figure 4 and figure 5. In both cases, in the plane strain region, the curves are grouped by the reference levels of the hardening exponent n. The same thing is observed in the left quadrant of the diagram. Here the larger influence on the limit strains is given by the anisotropy coefficient r_0. The scattering is more visible in the biaxial region. As expected, the larger limit strains are obtained by using Hill48 yield criterion. The width of the limit band is roughly the same for both constitutive models.

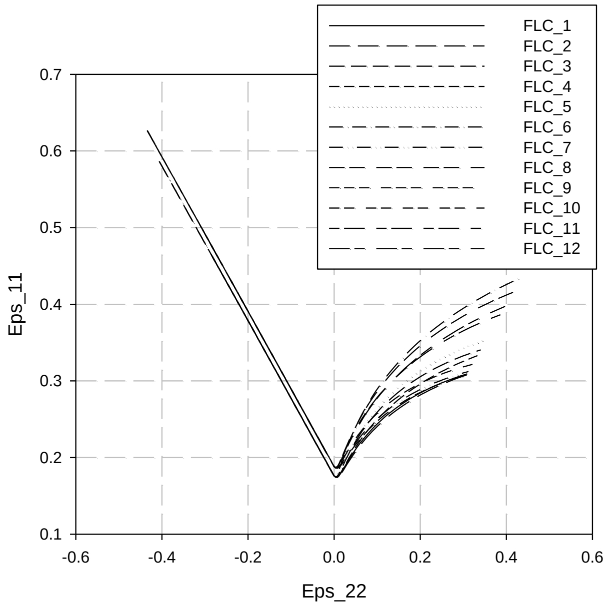

Fig.4. Forming limit curves obtained by varying the mechanical parameters using BBC2005 yield criterion

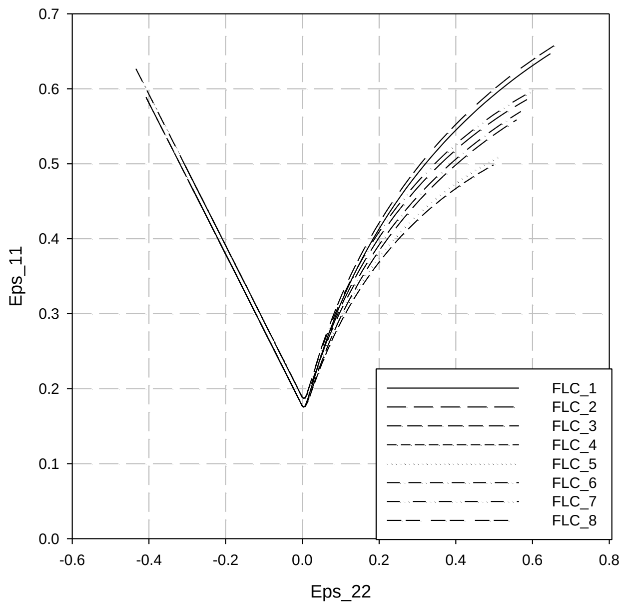

Fig.5. Forming limit curves obtained by varying the mechanical parameters using Hill48 yield criterion

6. CONCLUSIONS

In this paper the influence of the mechanical parameters on the forming limit curve has been studied using the Marciniak-Kuczinski necking model. The plasticity of the sheet metal has been described by Hill48 and BBC2005 yield criteria. The technique design of experiment has been used in order to optimize the number of simulations. By applying the ANOVA method, the percent contribution of each parameter on the limit strains has been calculated. In the case of Hill48 model with Swift's hardening law, only five mechanical parameters are taken into account, while in the case of BBC2005 yield criterion with Swift's hardening law, the influence of eight parameters is studied. It has been noticed that in the plane strain region of the FLC, the hardening exponent has a greater influence on the limit strains no matter each yield criterion is used. In the tension-compression region, the most influent is r_0 both yield criteria. Only in the right quadrant of the forming limit curve the scattering of limit strains is very significant. The limit strains obtained in the case of Hill48 are greater than in a BBC2005 yield criterion. The limit band obtained in the both cases has roughly the same width.

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