Theoretical Model for Forming Limit Diagram Predictions Without Initial Inhomogeneity

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Abstract.

We report on our attempts to build a theoretical model for determining forming limit diagrams (FLD) based on limit analysis that, contrary to the well-known Marciniak and Kuczynski (M-K) model, does not assume the initial existence of a region with material or geometrical inhomogeneity.

We first give a new interpretation based on limit analysis for the onset of necking in the M-K model. Considering the initial thickness defect along a narrow band as postulated by the M-K model, we show that incipient necking is a transition in the plastic mechanism from one of plastic flow in both the sheet and the band to another one where the sheet becomes rigid and all plastic deformation is localized in the band.

We then draw on some analogies between the onset of necking in a sheet and the onset of coalescence in a porous bulk body. In fact, the main advance in coalescence modelling has been based on a similar limit analysis with an important new ingredient: the evolution of the spatial distribution of voids, due to the plastic deformation, creating weaker regions with higher porosity surrounded by sound regions with no voids. The onset of coalescence is precisely the transition from a mechanism of plastic deformation in both regions to another one, where the sound regions are rigid.

We apply this new ingredient to a necking model based on limit analysis, for the first quadrant of the FLD and a porous sheet. We use Gurson's model with some recent extensions to model the porous material. We follow both the evolution of a homogeneous sheet and the evolution of the distribution of voids. At each moment we test for a potential change of plastic mechanism, by comparing the stresses in the uniform region to those in a virtual band with a larger porosity. The main difference with the coalescence of voids in a bulk solid is that the plastic mechanism for a sheet admits a supplementary degree of freedom, namely the change in the thickness of the virtual band. For strain ratios close to the plane-strain case the limit-analysis model predicts almost instantaneous necking but in the next step the virtual band hardens enough to deactivate the localisation condition. In this case we apply a supplimentary condition for incipient necking similar to the one used in Hill's model for the second quadrant. We show that this condition is precisely the one for incipient bifurcation inside the virtual (and weaker) band.

Finally we discuss some limitations, extensions and possible applications of the new necking model based on limit analysis.

Keywords: Forming limit curve; Sheet metal; Strain localization; Gurson's model
PACS: 81.20.Hy, 62.20.F, 81.40.Lm

INTRODUCTION

The Forming Limit Diagram (FLD) is used in sheet metal forming to predict the onset of necking during sheet processing. Necking or tearing starts when the plastic deformation, initially present over a large region of the sheet, becomes localized in a thin band, with a width comparable to the thickness of the sheet. Subsequent loading leads to a rapid decrease of the thickness of this band and the rupture of the sheet.

The key feature of the FLD is the forming limit curve (FLC), an empirically determined curve in the space of principal in-plane strains, that defines the boundary between safe strains, where no necking occurs, and unsafe strains, prone to necking and sheet rupture. Keeler and Backhofen [1] and Goodwin [2] proposed and developed the use of the FLD as a design tool: for a complex forming process, as long as the strain paths for each point of the sheet remain below the FLC (in the safe region), there will be no necking and sheet rupture. Usually, a supplimentary safety factor is considered by lowering the experimental FLC with some constant strain.

Implicit in this approach is the supposition that the FLC depends only on the sheet material properties and not on the strain path. However, recent developments exemplified by Graf and Hosford [3] have shown that the FLC can vary widely when a bilinear strain path is used instead of a linear path. This means that forming processes where the ratio of plastic strains varies signficantly throughout the process may need several FLCs to predict necking and failure. Several authors (see [4] and [5]) have proposed to use instead a stress-based limit curve and diagram, claiming that these are inherently more path-independent that their strain-based analogue.

We start by discussing the existing models for the onset of necking in thin sheets, divided into three classes: critical-value based models, models based on bifurcation of plastic flow in a homogeneous sheet and models based on a bifurcation analysis with some initial imperfection. Recent reviews of theoretical models for FLD may be found in [6], [7] and [8].

Considering the well-known Marciniak and Kuczyński (M-K) model [9] based on the postulated existence of an initial thickness defect along a narrow band, we show that its necking condition can be interpreted as a change of plastic mechanism in limit analysis. Precisely, before the onset of necking, the plastic mechanism is that of plastic deformation both in the band and in the uniform sheet, while at incipient or after necking, plastic deformation inside the band is accompanied by a rigid mechanism for the rest of the sheet.

We then draw on some analogies between the onset of necking in a thin sheet and the onset of coalescence in a porous bulk body. Similarly to the case of necking, the onset of coalescence has been modelled by some critical value theory [10], by plastic flow bifurcation analysis [11], [12] and by limit analysis [13]. Actually, the bifurcation analysis alone largely overestimates the onset of coalescence. In fact, the main advance in coalescence modelling has been based on limit analysis with an important new ingredient: the evolution of the spatial distribution of voids, due to the plastic deformation, creating weaker regions with more voids surrounded by stronger regions with less voids. The onset of coalescence is precisely the transition from a plastic mechanism with plastic deformation everywhere to another one where the stronger regions become rigid.

We explore this new avenue for the onset of localized necking. We assume that while a metal sheet is homogeneous at the macro level, it is probably inhomogeneous at the meso level (grains and grain boundaries, foreign inclusion, voids, etc.). While we still suppose that these inhomogeneities give by averaging an initially homogeneous structure, we take into account the change of their spatial distribution due to plastic deformation. For example, we expect that inclusions or voids that are initially equally spaced in the rolling and transverse directions, will, after some plastic deformation, be father apart in the major strain direction than in the minor strain direction. Once this spatial distribution becomes highly anisotropic, there appears the possibility of a potentially weaker band, perpendicular to the direction of major strain, that could trigger a transition in plastic flow mechanism: from a homogeneous plastic deformation in the entire sheet to one in which only the weak band deforms plastically while the surrounding regions become rigid.

We apply this new necking model based on limit analysis to a porous sheet. In ductile metals and alloys, voids appear during plastic deformation due to the cracking of foreign inclusions or a second fragile phase or to matrix/inclusion decohesion. These voids grow because of the plastic incompressibility of the surrounding matrix. The effects of voids and in general of damage on the limit strains predicted with the M-K theory have already been studied by Brunet *et al.* [14], [15], Hu *et al.* [16], Chow and Yang [17] and others (for a review see [7]. In general there is a consensus that while the voids show a relatively small growth during deformation of the sheet (due mainly to a low stress triaxiality present in a sheet), they become the predominant rupture mechanism in the neck region *a*fter the onset of necking. It is therefore rather unusual to choose precisely the voids as the culprit for necking in a uniform sheet; nevertheless we show that taking into account the evolving distribution of voids may explain necking without any initial localized defect.

We conclude by discussing limitations, extensions and possbile applications of the new sheet necking model based on limit analysis.

SHEET NECKING MODELS

Various theoretical models have been developed for predicting the forming limit curve. There is a fundamental difference between the first quadrant (positive minor strains) and second quadrant (negative minor strains) of the FLD, so that many theoretical models apply to only one quadrant.

Some theoretical models are based on a postulated maximal or critical value that once attained, gives rise to necking. The first such model was proposed by Swift [18] for diffuse necking and later developed by Hora [19] (so-called Modified Maximum Force Criterion). Bressan and Williams [20] proposed also a critical-value type model for the first quadrant, where necking is initiated in some through-thickness shear band when the shear stress on this band attains a critical value. A similar shear failure criterion was recently proposed by Lin *et al.* [21]. Another model based on a critical plastic deformation energy has been proposed by Chen *et al.* [22]. Generally, the critical value is determined from fitting the predictions of the model to one experimental point on the FLC, typically the plane strain point (zero minor strain).

Critical-value type models are relatively simple to implement and are widely used in the forming industry. One potential advantage of these models is that they could in principle be used directly in the finite-element simulation to predict incipient necking or failure for each each Gauss point, with no need for a forming limit diagram. However, this advantage is again conditioned by the strain-path independence of the proposed critical value.

Another family of models for the forming limit curve is based on a bifurcation analysis. In general, a bulk body made from a rate-independent elasto-plastic materials requires some *s*oftening behaviour in order to exhibit plastic flow localisation along narrow bands. This softening may be due to a temperature decrease of the yield stress, or to some damage mechanisms, etc. From the dynamical point of view, at incipient bifurcation the equations of movement loose elipticity in a well described manner and therefore admit stationary strain rate discontinuities, identified with narrow localization bands. In contrast, a sheet admits a very peculiar softening behavior of the total force acting on a section[1], mainly a decrease in the sheet thickness. It is therefore tempting to model the necking within sheets by a bifurcation analysis.

The first bifurcation analysis (and the first theoretical model for FLC) was proposed by Hill [23]. He considered a rigid-plastic sheet and was able to predict a bifurcation only in the second quadrant of the FLD, where there are in-plane directions with zero extension. On the contrary, in the first quadrant of positive principal in-plane strains, Hill's theory predicts infinite ductility with no necking, at variance with experimental data.

Two approaches were developed to cope with the first quadrant case. The first is due to Marciniak and Kuczyński [9] who postulated the existence of a thickness defect along a narrow band in an otherwise uniform sheet. When a constant strain ratio deformation is imposed on the uniform sheet, the strain state in the defective band changes gradually toward the plane strain mode (or zero extension mode inside the band), accompanied by a fast increase of the deformation rate of the band and a fast decrease of its thickness, similar to the experimental observed necking behavior. In the last decades, the M-K has been widely extended and used, see the review by Banabic [7].

An alternative approach is due to Stören and Rice [24], based precisely on previous work by Rudnicki and Rice [11] on plastic flow localisation in planar bands for soil and rock bodies. It assumes the development of a *c*orner on the yield surface, approximately modelled by the J_2 deformation theory. At such a vertex, there exist an entire range of strain rates normal to the yield surface. During deformation, this vertex becomes more and more pointed and the range of normals enlarges until a bifurcation in a band becomes possible. This approach also confirms that for a rigid-plastic material with a smooth yield surface, no necking bifurcation is possible in the first quadrant, except under conditions of plane strain. Inclusion of elasticity predicts some bifurcation but at very large strains and is therefore of no help.

We will later show that this last conclusion can be ammended when one takes into account the inhomogeneous distribution of voids created by the deformation and that a necking bifurcation is still possible in some region of the first quadrant of the FLD.

LIMIT ANALYSIS INTERPRETATION OF THE M-K MODEL

We will now show that the M-K model for the onset of necking admits a simple interpretation in terms of limit analysis or limit loads. Figure 1a shows the typical configuration of the homogeneous regions A along with the thickness defective region B. As it typical for the first quadrant of the FLD, we consider only the case where the region B is perpendicular to the direction of the major strain Ox_1. To simplify the discussion we also suppose that the coordinate system of Figure 1 is aligned with the rolling and transverse directions of the sheet, and therefore also with the orthotropy axes of the yield locus.

In the region A we impose a constant strain rate ratio:

$$\frac{d_{22}^A}{d_{11}^A} = \rho, \quad 0 \leq \rho \leq 1, \qquad d_{12}^A = 0. \tag{1}$$

where d is the strain rate and all variables carry a superscript showing the region they belong to. The last equation is implied by the condition that the strain rates d_{11}, d_{22} are principal strain rates.

Using the plastic normality condition and the condition of plane stress, eqs. (1) uniquely determine the stress σ^A for the region A[2].

[1] The total force is the product of the in-plane stress and the thickness.

[2] The alignement of the principal strains with the orthotropy axes of the sheet also implies that $\sigma_{12}^A = 0$

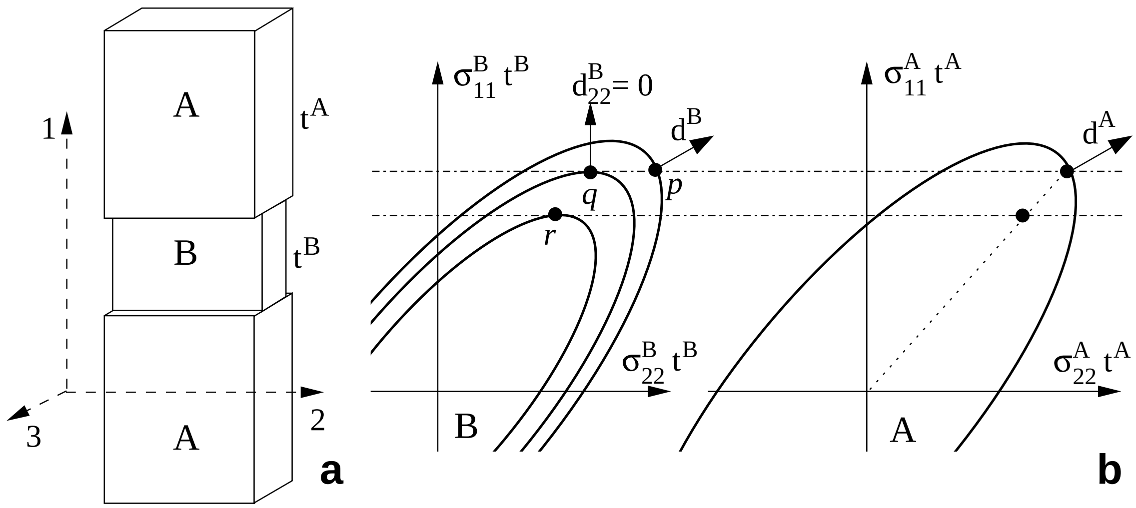

FIGURE 1. M-K model: thickness defect along a narrow band (**a**) and limit load interpretation of the onset of necking (**b**).

The equilibrium and compatibility conditions for the two regions are given by:

$$\sigma_{11}^A t^A = \sigma_{22}^B t^B, \qquad \sigma_{12}^A t^A = \sigma_{12}^B t^B = 0, \qquad d_{22}^A = d_{22}^B \tag{2}$$

where t is the actual thickness. These conditions also uniquely define the stress and strain rate for the region B.

In the usual approach to the numerical solution of the M-K model, these differential equations are integrated using an implicit Euler solver. During deformation, the strain rate in the region B rotates toward the plane strain condition where $d_{22}^B = 0$. Because of eq. (2_3), and the normality condition for region B, this implies that $d_{11}^B \to \infty$ when $d_{22}^B \to 0$. This in turn implies an infinitely fast decrease of the thickness in region B which is the M-K definition for incipient necking. In practice, the solver is stopped as soon as d_{11}^B / d_{22}^B becomes larger than a predefined value, usually 10.

We now propose an alternative explanation for the incipient necking predicted by the M-K model. In Figure 1b we have plotted the intersection of the yield loci for both regions A and B with the hyperplane $\sigma_{12} = \sigma_{13} = \sigma_{23} = \sigma_{33} = 0$. In order to impose the equilibrium condition or eq. (2_1), we have scaled the yield loci with the respective actual thicknesses. Starting with the known scaled stress $\sigma_{11}^A t^A$ in the region A, we seek the intersection of the line $\sigma_{22}^B t^B = \sigma_{11}^A t^A$ with the scaled yield locus of the region B. There are three posibilities, labeled with (p,q,r) in Figure 1b: two points of intersection, one tangent point, and no intersection.

For case (p), the two points have normals with different signs of d_{22}^B and therefore the correct choice is governed by eq. (2_3) and the sign of d_{22}^A. The second case (r) is precisely the onset of necking in the M-K model. As it easily inferred from Figure 1b, this case has two equivalent interpretations:

$$d_{22}^B = 0 \Longleftrightarrow \sigma_{11}^B = \sup\left(\sigma_{11} \mid \exists \sigma_{22}, \Phi^B(\sigma_{11}, \sigma_{22}) \le 0\right) \tag{3}$$

where Φ^B is the convex yield function for the region B. The second equation above shows that at the onset of necking the region B has reached its limit load for the x_1 direction. The third case (r) reinforces this limit-load interpretation: when there is no intersection, the equilibrium condition require that the stresses in region A are inside the convex yield locus. This means that region A is rigid, with no plastic deformation, and this implies:

$$d_{11}^A = d_{22}^A = 0, \qquad d_{22}^B = d_{22}^A = 0 \tag{4}$$

so that region B is necessarily in the plane strain condition. We observe that this last case is incompatible with an imposed strain rate ratio in region A as required by eq. (1_1); in this case the simplest solution is to switch to a constant stress ratio (shown with a dotted line in Figure 1b).

In conclusion, we have shown that the onset of necking in the M-K model is characterized by the attainment of a limit load for the defective region and a transition from a state of plastic deformation in both regions to one of plastic deformation inside the defective region and a rigid one inside the defect-free region.

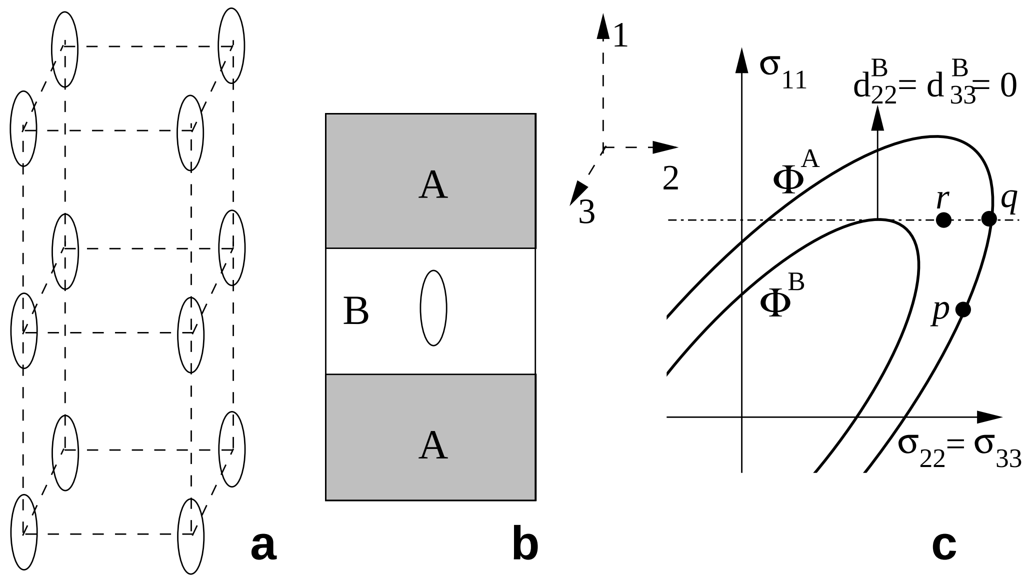

FIGURE 2. Typical coalescence model based on limit analysis: distribution of voids after some deformation (**a**), elementary cell showing sound and highly porous layers (**b**) and limit load interpretation of the onset of coalescence with stress states in the sound regions before coalescence *p*, at the onset of coalescence *q* and during coalescence *r* (**c**).

COALESCENCE MODELS FOR DUCTILE POROUS MATERIALS

Initially the voids in a ductile porous material grow due the incompressibility of the surrounding material. In their pioneering work, Koplik and Needleman [25] have numerically analyzed an elementary cell in a material with periodic voids, submitted to conditions of constant stress ratio (with axisymmetric loading and predominant axial stress); after some deformation the plastic flow becomes localized in the ligaments between the voids thus leading to an accelerated growth and subsequent coalescence of voids. An analytical model for the same elementary cell has been proposed by Gologanu *et al.* [26], based on a sandwich model with three layers - a highly porous one surrounded by two sound layers. There are two possible regimes - one with rigid outer layers and the other with plastic sound layers. The evolution of intervoid distances may trigger the rigid/plastic regime and therefore the onset of coalescence. Recently, Leblond and Mottet [27] have extended this analysis to the case of a combined axisymmetric and shear loading, treating within the same model the coalescence of voids and the formation of shear bands along voided sheets.

Independently, Thomason [13] has provided an analytical solution for the critical normal stress acting on a periodic planar array of rectangular voids where only the ligaments between voids are under plastic flow, the upper and lower blocks being rigid. He then used this particular solution to determine the onset of coalescence by the following limit analysis recipe: use a non-localized plastic flow solution (given by some homogeneized model for porous solids) as long as the normal stress given by this theory is below the critical stress; otherwise switch to the rigid blocks/plastic ligaments model.

Another successful model has been proposed by Perrin [12]. Similarly to the above models he follows the evolution of the distribution of voids and once a highly porous layer is formed, he applies to it the localized band bifurcation analysis of Rudnicki and Rice [11].

Figure 2 shows a typical coalescence model. The essential ingredients are the anisotropic distribution of voids due to the plastic deformation (**a**), the consideration of the horizontal sound layers *A* and highly porous layer *B* (**b**) and finally the limit load interpretation for the onset of coalescence, completely analogous to the one in Figure 1b pertaining to the onset of necking in the M-K model: when stresses in the sound region attain the maximal stress supported by the porous layer at *q*, there is a change in plastic regime toward a rigid behaviour of the sound regions (stresses in the sound region are inside the yield locus Φ^A at *r*) while the porous layer remains in a strain-state compatible with this rigid behaviour $d^B_{22} = d^B_{33} = 0$.

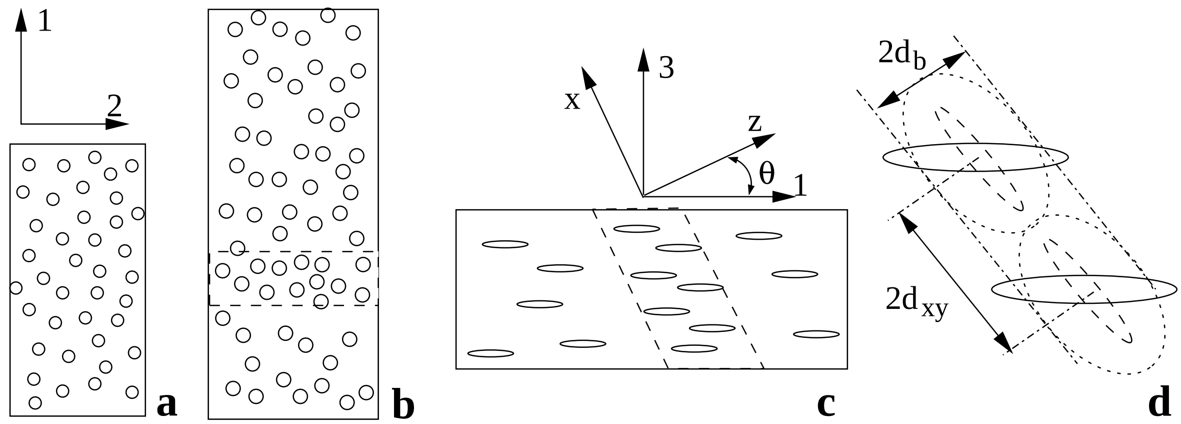

FIGURE 3. LA necking model: Spherical voids with an initial homogeneous distribution (**a**), in plane virtual localisation band after a plastic deformation with vertical major strain (**b**), through-thickness virtual localisation band after a biaxial deformation that rendered the voids oblate (**c**), method for determining the increased porosity in a virtual localisation band (**d**).

NECKING MODEL BASED ON LIMIT ANALYSIS FOR POROUS SHEETS

Based on the observed analogy between the onset of sheet necking as predicted by the M-K model and the onset of coalescence of voids in a porous bulk solid, we develop now a new necking model without an initial imperfection based on limit analysis (LA).

Let us consider a porous sheet with a matrix obeying a rigid-plastic law with von Mises yield criterion. An initial isotropic distribution of voids (Fig. 3a) will evolve into an anisotropic distribution after some deformation of the sheet (Fig. 3b).

We model the voided sheet using the ellipsoidal Gurson model from [28] without taking into account the distribution of voids. At each moment we test for localisation inside virtual bands with various normals n (Fig. 3c). For this test we do take into account the anisotropic distribution of voids that leads to an increased porosity inside the band. We first need to determine the mean void interspacings $2d_{xy}$ and $2d_z$ in the plane of the band and in the perpendicular direction z, parallel to n; then we need to estimate the thickness of the band $2d_b$ or equivalently the ratio $c = d_b/d_z$ and finally we need a model for the limit load the band can still sustain.

Let us denote F the deformation gradient at the actual time. By assuming that the void interspacings are governed by the evolution of some elementary area and length, Leblond and Mottet [27] were able to determine an expression for the ratio $r \equiv d_{xy}/d_z$. It is easy to generalize their result to the case of a distribution of voids that has already been submitted to some deformation gradient F_0 prior to the analyzed deformation process, again starting from an isotropic distribution:

$$r \equiv \frac{d_{xy}}{d_z} = \sqrt{\det F F_0}\left(nFF_0F_0^TF^Tn\right)^{-3/4}. \tag{5}$$

where n is the normal to the band (parallel to direction z).

The choice of the band thickness for coalescence models has been widely discussed in the literature. For example Thomason's model is based on the choice $2d_b$ equal to the void height in the direction n, in order to best model plastic flow localisation in the ligaments between voids. Based on experimental observations on sheet rupture showing that necking in general precedes void coalescence, we follow here the proposal of Perrin [12] and Gologanu *et al.* [26] and choose the thickness d_b such that the resulting elementary cell surrounding the void is the best possible approximation for an ellipsoid confocal with the void. This choice is compatible with using an ellipsoidal Gurson model for determining the limit load of the band; this model needs the porosity f^p and the shape factor S^p inside the band.

A supplementary difficulty appears if the band is not parallel to the void axes or if the void is not axisymmetric in the plane of the band. In this case, we determine an equivalent axisymmetric void by the following recipe: we project the initial void onto the plane xy to obtain an ellipse, we replace this ellipse with a circle of radius a_{xy} of same area and we determine the height a_z of the new void by imposing equal volumes for the original and new voids. In the sequel we will need only the following special case: the initial void is aligned with the sheet axes and has semiaxes a_1, a_2, a_3

and the normal to the band is given by $n = (n_1, 0, n_3)$:

$$a_{xy} = a_2^{1/2}\left(a_3^2 n_1^2 + a_1^2 n_3^2\right)^{1/4}, \qquad a_z = \frac{a_1 a_3}{\left(a_3^2 n_1^2 + a_1^2 n_3^2\right)^{1/2}}, \qquad \exp(S^p) = w^p = \frac{a_z}{a_{xy}}. \tag{6}$$

We note that for a initial void that is also axisymmetric (either prolate or oblate), the last equation defines uniquely the shape factor S^p of the projected void as a function of the initial shape factor S.

The confocality condition and the porosity are given now explicitly by:

$$d_b^2 - d_{xy}^2 = a_z^2 - a_{xy}^2, \qquad f = \frac{\frac{4\pi}{3} a_{xy}^2 a_z}{8 d_{xy}^2 d_z} \tag{7}$$

Using eqs. (5, 6, 7) we obtain the final result for the porosity f^p inside the band:

$$c = \frac{f}{f^p} = \left[r^2 + \left(\frac{6}{\pi} f r^2 w^2\right)^{2/3} \left(1 - \frac{1}{w^2}\right)\right]^{1/2}. \tag{8}$$

For a spherical void, this expression reduces to $c = r$ as proposed by Leblond and Mottet [27].

We still need to provide an expression for the limit load of the virtual localization band. Let σ^p be the stress on the inclined band in Figure 3d due to stress equilibrium and strain compatibility with the uniform sheet:

$$\sigma_{zz}^p = \sigma_{11}\cos^2\theta, \qquad \sigma_{xz}^p = \sigma_{11}\sin^2\theta, \qquad \sigma_{yz}^p = 0, \qquad d_{yy}^p = 0. \tag{9}$$

We note that at variance with coalescence models we do not impose $d_{xx}^p = d_{xy}^p = 0$ but use rather $\sigma_{xx}^p = \sigma_{xy}^p = 0$ resulting from plane stress conditions. Let $\Phi^p(\sigma, f^p, S^p) = 0$ be the yield surface of the porous band, where we have omitted the dependence on other state parameters. Then the limit load problem for the band can be written as:

$$\alpha_{max} = \sup\left\{\alpha \mid \Phi^p(\alpha\sigma^p, f^p, S^p) \leq 0, \frac{\partial\Phi^p}{\sigma_{22}} = 0\right\} = 1. \tag{10}$$

There is no analytical closed-form solution of this equation; we solve it numerically using a Lagrangean formulation that will be described elsewhere.

There is a supplementary condition for incipient necking that has generally be neglected in coalescence studies. The attainment of the limit load in the band is not sufficient, as the subsequent deformation of this band and increased hardening inside the band may instantenously deactivate the limit load condition. For a vertical through thickness band this new condition is simply:

$$\widehat{\dot{\sigma}_{11}^p t} \leq 0, \qquad \dot{\sigma}_{11}^p + \sigma_{11}^p d_{33}^p \leq 0. \tag{11}$$

where the derivatives must be taken for a porous band that remains compatible with rigid blocks outside the band. The last equation is similar to the one used by Hill for the second quadrant and identical to eq. (7) in Stören and Rice [24] for the bifurcation along a band perpendicular to the major strain axis, without any consideration of a vertex on the yield surface. This shows that the new model contains as a special case the bifurcation theory of Stören and Rice, applied not to the uniform sheet but to a virtual band with increased porosity.

We do not present here the supplementary condition for an inclined band as we have found that it is always preceded by the limit load condition for strain pathes close to the biaxial one, exactly where we expect that inclined bands may be first to localize.

The final model we use is that for a non-inclined band incipient necking is attained when both conditions are true:

$$\alpha_{max} \leq 1, \qquad \dot{\sigma}_{11}^p + \sigma_{11}^p d_{33}^p \leq 0. \tag{12}$$

while for an inclined band only the first condition is used.

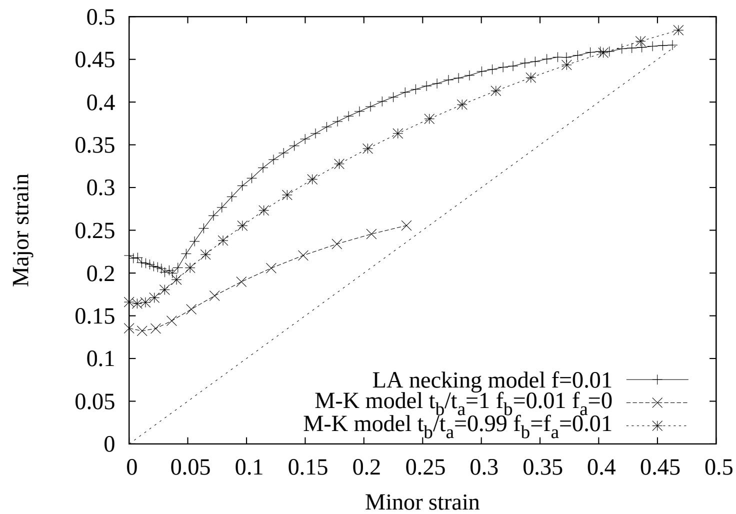

FIGURE 4. Numerical FLD predictions for a spherical Gurson model: LA necking model versus M-K model.

NUMERICAL RESULTS

We first consider the simplest possible model for a porous sheet with a matrix having a rigid-plastic behavior with von Mises yield surface, Swift hardening with $K = 417MPa$, $\varepsilon_0 = 0.01$ and a hardening exponent $n = 0.245$ and an initial porosity $f_0 = 0.01$. We also suppose that voids are initially spherical and remain so during sheet deformation. Figure 4 compares the prediction of the new LA necking model with the M-K model. The LA necking model shows two different regions: the first region, close to the plane-strain conditions, is one where the bifurcation condition eq. (11) is attained before the limit load condition eq. (10), while for the second region the reverse is true. For this spherical Gurson model the limit load in an inclined band was attained always after it was attained in the non-inclined band. The M-K model results shown on the same figure correspond to the case of an initial damage imperfection without thickness imperfection - the region B is porous but region A is sound, while the other curve corresponds to the dual case where the porosity is the same in both regions but there is an initial thickness imperfection with thickness ratio 0.99. We now consider the same porous material as before but we let the shape of the voids evolve toward oblate ellipsoids. Figure 5 compares again the results of the new LA necking model and M-K models. In this case one observes that there appears a third region around the biaxial strain condition, where some inclined band attains the limit load before the non-inclined band.

CONCLUSION

We have shown that the evolution of the spatial distribution of voids in a porous sheet may lead to the formation of weak bands with higher porosity and a low limit load compatible with a rigid behavior of the rest of the sheet. We were thus able to predict the onset of localized necking in the first quadrant of the FLD without any initial imperfection. We predict a Hill type incipient necking close to plane strain, an inclined (throgh thickness) necking band close to biaxial strain, and a non-inclined band in between.

We do not expect that voids will explain the FLC for all materials; nevertheless we propose that the evolution of the spatial distribution of some other defects may also trigger a similar limit load necking mechanism. The biggest advantage of a model without initial inhomogeneity is that it can be directly included in finite element models by following at each Gauss point some internal parameters describing the evolution of the voids and comparing at each time step the stresses with limit loads corresponding to incipient necking bands with some arbitrary orientation.

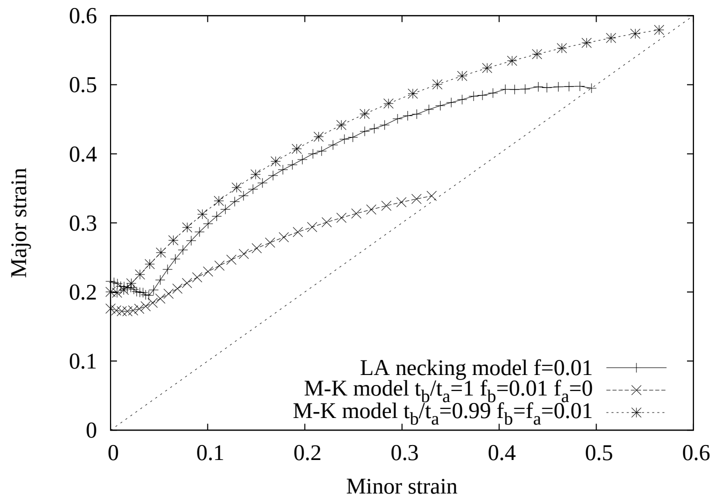

FIGURE 5. Numerical FLD predictions for an ellipsoidal Gurson model: LA necking model versus M-K model.

ACKNOWLEDGMENTS

This work was supported by the National Council of Scientific Research, in the frame of the Project PCCE 100/2010.

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