Abstract. First, the author contributed with some aspects concerning the chronology of the developments of the Marciniak model. A review of the recent developments in the last decade of Marciniak-Kuczynski model is presented in the paper. Implementation of the new constitutive and polycrystalline models, enhancing the existing models to take into account new material, process parameters and strain-paths, modeling the Forming Limit Band concept are briefly reviewed. Capabilities of some commercial programs specially designed for the computation of forming limit curves (FLC) are also analyzed.

Keywords: Formability, Forming Limit Diagrams, Marciniak-Kuczynski theory.

1 Introduction

Formability describes the capability of a sheet metal to undergo plastic deformation in order to get some shape without defects. During the last decades different assessment methods of metals sheets formability have been developed. The most useful tool used to assess formability is the forming limit diagram (FLD). It has been almost 50 years since this concept was published by Keeler (1961; 1963) and then developed by Goodwin (1968) for the right side of the diagram. This method meets both manufacturer and user’s requirements and is widely used in factory and research laboratories. One of the major advantages of the FLD concept is that the plastic instability can also be described by theoretical models. A detailed presentation of this method can be found in the literature (Banabic, 2000a; Banabic, 2000b; Banabic et at. 2007; Banabic, 2010; Hora & Krauer, 2006; Wagoner et at., 1989; Xu, 2006).

Various theoretical models have been developed for the calculation of forming limit curves (FLC). The first ones were proposed by Swift (1952) and Hill (1952) assuming homogeneous sheet metals (the so-called models of diffuse necking and localized necking),
respectively). The Swift model has been developed later by Hora (so-called Modified Maximum Force Criterion - MMFC) (Hora & Tong, 1994; Hora et al., 1996; Hora & Tong, 2008). Marciniak (1965) proposed a model taking into account that sheet metals are non-homogeneous from both the geometrical and the microstructural point of view. Stören and Rice (1975) have been developed a model based on the bifurcation theory. Dudzinski and Molinari (1988) used the method of linear perturbations for analyzing the strain localization and computing the limit strains. Bressan and Williams (1983) have introduced so-called “Through Thickness Shear Instability Criterion” in order to take into account the shear fracture mode. Based on the analysis of the influence of the stress distribution through the thickness on the mode of failure, Stoughton (2000) has proposed a generalized failure criterion. Since the theoretical models are rather complex and need a profound knowledge of continuum mechanics and mathematics while their results are not always in agreement with experiments, some semi-empirical models have been developed in recent years. The models used for FLC prediction are presented in detail (formulation of the model, solving methods, numerical aspects, advantages and limitations) in the book (Banabic, 2010).

2 A briefly presentation of the Marciniak-Kuczynski model

Shortly after the publishing of the Forming Limit Diagram concept, on the basis of the experimental investigations concerning the strain localization of some specimens subjected to hydraulic bulging or punch stretching, Marciniak (1965) and Marciniak and Kuczynski (1967) developed a limit curve prediction model. This model is based on the hypothesis of the existence of imperfections in sheet metal. According to Marciniak’s hypothesis, sheet metal has, from manufacturing, geometrical imperfections (thickness variation) and/or structural imperfections (inclusions, gaps). In the forming process these imperfections progressively evolve and the plastic forming of the sheet metal is almost completely localized in them, leading to the necking of the sheet metal. The realism of this hypothesis has been experimentally shown by Azrin and Backofen (1970). This model has been intensely used and developed by researchers due to the advantages it offers: it has an intuitive physical background; it correctly predicts the influence of different process or material parameters on the limit strains; the predictions are precise enough; the model can be easily coupled with Finite Element simulation software for sheet metal forming processes. The main drawbacks of this model are: the prediction results are very sensitive to the constitutive equations used, as well as to the values of the non-homogeneity parameter; in the case of advanced material models, the equation system of the model is quite difficult to
solve and lacks robustness.

A few years later, Marciniak (1968) made a deep analysis of the strain localization phenomenon from the right side of the FLD and extended his initial model to cover this area. The models have periodically been brought in discussion by specialists in dedicated symposia (see Koistinen & Wang, 1978; Hecker et al. 1978; Wagoner et al. 1989; Hora & Krauer, 2006;) or in special sections in conferences (NUMISHEET, NUMIFORM, IDDRG, ESAFORM etc.). Further developments of the Marciniak limit curve prediction models are synthetically described in the review paper (Banabic et al., 2010).

On the basis of experimental investigations concerning strain localization, it was concluded that necking is usually initiated by a geometrical or structural non-homogeneity of the material (Marciniak, 1965). The analysis of the necking process has been performed assuming a geometrical non-homogeneity in the form of a thickness variation. This variation is usually due to some defects in the technological procedure used to obtain the sheet metal. The thickness variation is generally gentle. However, the theoretical model assumes a sudden variation in order to simplify the calculations (Figure 1). The theoretical model proposed by Marciniak assumes that the specimen has two regions: region “a” having a uniform thickness \( t_0^a \), and region “b” having the thickness \( t_0^b \). The initial geometrical non-homogeneity of the specimen is described by the so-called “coefficient of geometrical non-homogeneity”, \( f \), expressed as the ratio of the thickness in the two regions: \( f = t_0^b / t_0^a \). In the MK model, the strain and stress states in the two regions are analyzed and the principal strain \( \varepsilon_1^b \) in region “b” in relation with the principal strain \( \varepsilon_1^a \) in region “a” is monitored. When the ratio of these strains \( \varepsilon_1^b / \varepsilon_1^a \) becomes too large (infinitely large in theory, but greater than 10 in practice), one may consider that the entire straining of the specimen is localized in region “b”. The shape and position of the curve \( \varepsilon_1^a - \varepsilon_1^b \) depend on the value of the \( f \)-coefficient. If \( f = 1 \) (geometrically homogeneous sheet), the curve becomes coincident with the first bisector. Thus this theory cannot model the strain localization for geometrically homogeneous sheets. The value of the principal strain \( \varepsilon_1^a \) in region “a” corresponding to non-significant straining of this region as compared to region “b” (the straining being localized in region “b”) represents the limit strain \( \varepsilon_1^{a*} \). This strain together with the second principal strain \( \varepsilon_2^{a*} \) in region “a” define a point belonging to the FLC. Assuming different strain ratios \( \rho = d\varepsilon_2 / d\varepsilon_1 \), one obtains different points on the FLC. Spanning the range \( 0 < \rho < 1 \), one gets the FLC for biaxial tension (\( \varepsilon_1 > 0, \varepsilon_2 > 0 \)). In this domain, the orientation of the geometrical non-homogeneity with respect to the principal directions is assumed to be the same during the entire forming process. A detailed analysis
of the Marciniak-Kuczynski model (formulation, solving methods, influence on the localization of the deformations etc.) is presented in the book (Banabic, 2010).

![Geometrical model of the Marciniak-Kuczynski theory](image)

**Fig. 1 Geometrical model of the Marciniak-Kuczynski theory**

### 3 Developments of the Marciniak-Kuczynski model

During the last decade the research in the field of the forming limits prediction using Marciniak-Kuczynski model have been focused mainly on the following aspects.

#### 3.1 Implementation of new constitutive equations in the models used for the computation of the limit strains

The results of the FLC prediction depend crucially on the constitutive equation of the material analyzed. The effect of the shape of the yield locus on the limit strains has been analyzed in detail by Barlat and Lian (1989). As we have emphasized in Banabic (2010), a lot of new yield criteria have been developed during the last decade. Many of those criteria have been already implemented in the computational models of the limit strains, in order to improve the predictive capabilities. Banabic have implemented various yield criteria in the MK model. For example he implemented Hill ’93 (Hill, 1993) yield criterion (Banabic, 1999; Banabic & Dannenmann, 2001); BBC yield criteria (Banabic et al., 2004a; Banabic, 2004; Paraianu et al. 2006); Cazacu-Barlat (Cazacu & Barlat, 2001) (Banabic et al., 2005b, Paraianu & Banabic, 2005; Paraianu et al. 2006). In figure 2 (Banabic, 2004) is presented the theoretical FLC predicted using BBC2003 criterion (Banabic et al., 2003) versus experimental data for AA5182-0 aluminium alloy. Mattiasson and Sigvant have analyzed in
a intensive program the influence of the yield locus shape on necking prediction (Mattiasson et at., 2007; Mattiasson et at., 2008; Mattiasson & Sigvant, 2008). Butuc used the Barlat (1997) (Butuc et at., 2002; Butuc et at., 2005; Butuc et at., 2006) and BBC2000 (Butuc et at., 2002) yield criteria. Cao and her coworkers (Cao et at., 2000; Yao & Cao, 2001) used the Karafillis and Boyce yield criterion (Karafillis & Boyce, 1993) in the MK model to analyze the effect of changing strain-paths on the FLC. Kuroda and Tvergaard (2000a) used four different yield criteria to fit a set of experimental data. Yld 2000 formulation (Barlat et at., 2000) has been included by Aretz (2004) in the MK model for studying the influence of the biaxial coefficient of plastic anisotropy on the FLCs. Kim et at. (2003) used the YLD2000 (Barlat et at., 2000) criterion to analyze the formability of a sandwich sheets. FLD for multi-layered sandwich sheets considering the material properties of each layer has been formulated with assumption of the visco-rigid plastic material based on the modified MK model (Kim et at., 2008). The anisotropic strain-rate potential was utilized for the plastic behavior of each layer. Vegter (Vegter et at., 1999; Vegter et at., 2008) have implemented their own yield criterion (Vegter & Boogaart, 2006) in the MK model. Ganjiani and Assempour (Ganjiani & Assempour, 2007a; Ganjiani & Assempour, 2007b; Ganjiani & Assempour, 2008) have improved the analytical approach for determination of FLC considering the effects of yield functions (Hosford, 1979; Karafillis & Boyce, 2003; Banabic et at., 2003). The Teodosiu hardening model (Teodosiu & Hu, 1995) associated with different yield criteria has been implemented by Butuc et at. (2003) and Haddag et at. (2008) in the MK theory for studying the influence of the loading path change on the limit strains. The effect of BBC2003 (Banabic et at., 2003) yield surface on the prediction of FLCs and the number of experimental anisotropy parameters on the accuracy of yield functions are studied by Ahmadi et at. (2009). The polynomial yield function developed by Soare et at. (2007) has been implemented in the MK model (Soare & Banabic, 2008) and has been used to analyze the sensitivity of the MK model to the shape of the yield surface (Soare & Banabic, 2009).

3.2 Implementation of the polycrystalline models

Houtte et al. (1994) has been implemented in FLC models (Van Houtte, 2005), the results being compared both with those provided by phenomenological models and with experimental data (Banabic, 2004). Van Houtte model (1994) coupled with a dislocation based hardening model (Teodosiu & Hu, 1995) have been implemented by Hiwatashi et al. (1998) and Van Houtte (2005) in order to predict the forming limits corresponding to change strain paths. A microstructural model developed for the description of the aluminium alloy hardening (ALFLOW) has been used by Berstad et al. (2004) to predict the forming limits of the AA3103-0 alloy. Boudeau et al. (1998), Boudeau and Gelin (2000) used the linear stability analysis combined with a polycrystalline model to predict and to analyze the influences on the FLC. A polycrystal plasticity model has been used by McGinty (2004) to conduct parametric studies of FLC. Knockaert et al. (2000) have used a rate-independent polycrystalline plasticity to predict the limit strains. The influence of the texture on the FLCs has been studied by Kuroda (2005) and Fjedbo et al. (2005). More recently, Signorelly et al. (2009a), (2009b), John Neil and Agnew (2009) have analyzed the forming-limit strains using a rate-dependent plasticity, polycrystal, self-consistent (VPSC) model, in conjunction with the Marciniak–Kuczynski (M–K) approach.

![Fig. 2 Theoretical FLC versus experimental data for AA5182-0 aluminium alloy](image)

### 3.3 Implementation of the ductile damage models

Several types of ductile damage models have been developed during the time, e.g. Gurson, Kachanov, Chaboche, Gologanu (see details in (Lemaitre, 2001)). Those models have been frequently used during the last decade for the computation of the limit strains. Brunet et al. have used the Gologanu model (Gologanu et al., 1997) for calculating such
limit strains (Brunet et al., 2001; Brunet et al., 2002; Brunet et al., 2005). The effects of texture and damage evolution on the limit strains have been studied by Hu et al. (1998). Chow et al. have developed a ductile damage model and implemented it into the MK theory both for linear (Chow et al., 1997) and complex load paths (Chow &Yang, 2001; Chow et al., 2001). An anisotropic model of Gurson type has been used by Huang et al. (2000) for the computation of the FLCs. Ragab et al., (2002) use a new model to predict the FLC for kinematically hardened voided sheet metals. Han and Kim (2003) used an original ductile fracture criterion to calculate the FLC. Lemaitre’s ductile damage model has been also implemented by Teixeira (2006). Parsa et al. (2009) have determined the Forming Limit Curves for of sandwich sheet using the Gurson damage model.

3.4 Enhancing the existing models to take into account new material or process parameters

The influence of different parameters on the limit strains has been analyzed since the end of the 1960’s. More recently, several new introduced parameters have been included in the MK and MMFC models: the shape of the yield locus (Banabic & Dannenmann, 1999), the forming temperature (Abedrabbo, 2006), (Hora et al., 2007),(Krauer et al., 2008), (Zhang et al., 2008) and the coefficient of biaxial anisotropy (Aretz, 2006). The influences of the different effects on the limit strains have been studied: the effect of the surface defects (Hiroi & Nishimura, 1997) the effect of the void growth (Ragab & Saleh, 2000), the effect of grain size (Shakeri, 2000). Chan (2003) has developed a model of forming limits prediction for the superplastic forming. Predictive models of localized necking for strain-rate-dependent sheet metals have been developed by Mattiasson et al. (2007), Mattiasson et al. (2008), Zhang et al. [313], Jie et al. (2009). The effect of the normal pressure on the formability of sheet metals is well known and has already been used in industry for a long time (Keeler, 1970). An analysis of sheet failure under normal pressure without assuming ductile damage has been done in the last period. Such an analysis was performed by using Swift-Hill models by Gotoh et al. (1995), Smith et al. (2003) and Matin and Smith (2005). Recently, Banabic and Soare (2008), Wu et al. (2008) and Alwood and Shouler (2008) have analyzed independently the influence of the normal pressure on the Forming Limit Curve using an enhanced MK model. The experimental researches of the Single Point Incremental Forming (SPIF) (Alwood et al. 2007; Jeswit & Young, 2005; Petek &Kuzman, 2007; Shim & Park, 2001) showed that the formability of the sheet in this process increases (the FLC is beyond the traditional FLC). Alwood et al. (2007) and Jackson and Alwood (2009) have
suggested that Through Thickness Shear influences formability in SPIF process. Based on these observations, Eyckens extend the MK model to analyze the influence of the Through Thickness Shear on the FLC (Eyckens et al., 2008; Eyckens et al., 2009; Eyckens et al., 2010).

3.5 Extending the FLC models for non-linear strain-paths

During the sheet metal forming processes, the material is usually subjected to complex strain patterns. Nakazima et al. (1971) has proved that complex loads modify the shape and position of the FLC’s. This fact imposes the determination of the limit strains for complex strain-paths. The development of the computational models for complex strain-paths in the frame of the MK theory has become an active research field in the early 1980’s (see Barata and Jalinier, 1984; Barata et al., 1985; Wagoner et al., 1989). The refinement of those models has been intensively approached only during the last period. Butuc et al. (2002a); Butuc et al. (2002b); Butuc et al. (2005); Butuc et al. (2006) has developed a general computer code for the FLC computation in the case of complex load paths using various hardening models (both phenomenological – Swift, Voce, and microstructural ones – Teodosiu-Hu). Rajarajan et al. (2005) have validated the CRACH model for the case of complex strain-paths. Cao et al. (2000), Yao and Cao (2002) analyzed the influence of the changing strain paths on the limit strains. Hiwatashi et al. (1998) have used Teodosiu’s model for studying the influence on the strain-path change on FLCs. Kuroda and Tvergaard (2000b) have studied the effect of the strain-path change on the limit strains using four anisotropic models.

3.6 Using advanced numerical methods for the solution of the limit strain models

Wagoner and his co-workers have used the finite element method for the numerical determination of the limit strains in the frame of the MK theory (Narashima & Wagoner, 1991; Zhou & Wagoner, 1991). Later on, FEM has been also used by Horstemayer et al. (1994), Tai and Lee (1996), Nandedkar and Narashima (1999), Gänser et al. (2003), Evangelista et al. (2002), Van der Boogaard and Huetink (2003), Lademo et al. (2004), Lademo et al. (2005), Berstad et al. (2004), Brunet et al. (2005), Paraianu and Banabic (2005), Teixeira et al. (2006), Hopperstad et al. (2006). The results reported by the researchers previously mentioned are promising.
3.7 Modeling the Forming Limit Band concept

The first results on the influence of the variability of the material parameters on the Forming Limit Curves have been reported by van Minh et al. (1973). Karthik et al. (2002) have studied the coil-to-coil, test-to-test and laboratoty-to-laboratory variability of sheet formability using OSU formability test. On the basis of the variability of the limit strains established by experiments (Carleer & Sigvant, 2006; Rechberger & Till, 2004), Janssens et al. (2001) introduced the Forming Limit Band concept. This is a strip containing almost all of the limit strain states. The concept has been extended by Strano and Colosimo (2006a; 2006b). Assuming the variability of the mechanical parameters of the sheet metal, Banabic and Vos (2007) and Vos and Banabic (2007) have developed a computational method of the Forming Limit Band. In the figure 3 is presented the predicted

![Predicted Forming Limit Band versus experimental data for AA6111-T43 aluminium alloy (LFLC-lower FLC, UFLC-upper FLC).](image)

Fig 3 Predicted Forming Limit Band versus experimental data for AA6111-T43 aluminium alloy (LFLC-lower FLC, UFLC-upper FLC).

Forming Limit Band versus experimental data for AA6111-T43 aluminium alloy. A new model based on the assumption of the thickness variations of the sheet (modeled by use of random fields) to predict the Forming Limit Band has been proposed by Fyllingen et al. (2009). An approach to statistically evaluate the forming limit in hydroforming processes when taking into account the variations in the material parameters has been reported recently by Kim et al. (2009).

3.8 Developing commercial codes for FLC computation

In the last decade, more commercial programs for the limit strains prediction have been
developed. In this section the most significant ones are presented.

Based on a Marciniak-Kuczynski mode, Banabic (2006) and Jurco and Banabic (2005) have developed so-called FORM-CERT commercial code. The BBC 2005 yield criterion (Banabic et al., 2005a) is implemented in this model. This yield criterion can be reduced to simpler formulations (Hill 1948; Hill 1979; Barlat 1989). In this way, the yield criterion can be also used in the situations when only 2, 4, 5, 6, or 7 mechanical constants are available. The program consists in four modules: a graphical interface for input, a module for the identification and visualization of the yield surfaces, of the strain hardening laws and a module for calculating and visualizing the forming limit curves. The numerical results can be compared with experimental data, using the import/export facilities included in the program. The FORM-CERT code can be directly coupled with the finite element codes.

Using the CRACH algorithm (based on the Marciniak-Kuczynski model), Gese and Dell (2006) have developed two software: CrachLAB, a product for prediction of the initial FLC and CrachFEM a product for coupling with the FEM codes. Criteria for ductile and shear fracture have been included in CrachFEM to cover the whole variety of fracture modes for sheet materials. The material model used to calculate instability describes: the initial anisotropy (using Hill (1948) and Dell et al. (2008) models), the combined isotropic-kinematic hardening and the strain rate sensitivity Dell et al. (2008). CrachFEM is now included in the FEM codes PamStamp and PamCrash of ESI Group.

4 Conclusions

In the past, the FLC models provided an approximate description of the experimental results. Such models were used especially for obtaining qualitative information concerning the necking/tearing phenomena.

At present, the FLC models allow a sufficiently accurate prediction of the limit strains, but each model suffers from its own limitations. There is no model that can be applied to any sort of sheet metal, any type of crystallographic structure, any strain- path or any variation range of the process parameters (strain rate, temperature, pressure, etc.).

The future research will be focused on a more profound analysis of the phenomena accompanying the necking and fracture of the sheet metals. On the basis of the analysis, more realistic models will be developed in order to obtain better predictions of the limit strains. New models will be developed for prediction of the limit strains for special sheet metal forming processes: superplastic forming, forming at very high pressure, incremental forming etc. Commercial codes allowing the quick and accurate calculation of the FLC’s
both for linear and complex strain-paths will be developed. The texture models will be also implemented in such commercial programs. The FLC computation will be included in the finite element codes used for the simulation of the sheet metal forming processes. The aim is to develop automatic decision tools (based on artificial intelligence methods) useful in the technological design departments. The stochastic modeling of the FLC’s will be developed in order to increase the robustness of the sheet metal forming simulation programs. More refined, accurate and objective experimental methods for the experimental determination of the limit strains (e.g. methods based on thermal or acoustic effects) will be also developed.

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**References**


methods in prediction of forming limits in sheet forming and tube hydroforming processes, ETH Zürich, Zürich, 37-42.


Boudeau, N., Gelin J.C., Salhi, S., 1998, Computational prediction of the localized necking in sheet forming based on microstructural material aspects, Computational Materials Science, 11, 45-64.


